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ABSTRACT

This paper focuses on radioactive practicle counting statistics in laboratory and field applications, intended to aid the Health Physics technician's understanding of the effect of indeterminant errors on radioactive particle counting. It indicates that although the statistical analysis of radioactive disintegration is best described by a Poisson distribution, the dead time of a counting instrument renders the use of the Poisson ineffective without correction for the gap distribution. A modification of Poisson statistics through the binominal limit is then shown to allow legitimate application of a Gaussian distribution in standard laboratory application. The evolution of Poisson probability is summarized and two relative activity formulas are derived from their Poisson origin through the final working formula. (Author/CS)

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MODIFICATION OF POISSON DISTRIBUTION IN RADIOACTIVE PARTICLE COUNTING

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ABSTRACT

The statistical analysis of radioactive disintegration is best described by a Poisson distribution. In actual practice, the dead time of a counting instrument renders the use of the Poisson ineffective without correction for the gap distribution. These corrections are mathematically rigorous, making them impractical in Health Physics field situations. Modification of Poisson statistics through the binomial limit allows legitimate application of a Gaussian distribution in standard laboratory applications. The evolution of Poisson probability is summarized and two relative activity formulas are derived from their Poisson origin through the final working formula.

TEXT

By necessity, radioactive counting statistics are a modified blend of two very different methods of statistical thought. Poisson probability most accurately describes the decay of radioactive isotopes (Diamond, 1964). However, there are mathematical problems inherent with the combined use of Poisson probability and some common particle counting methods which make strict adherence to Poisson methods infeasible.

A common Health Physics solution for this problem is to combine Poisson and Gaussian statistics into workable formulas for the laboratory. To fully understand the utilization of this modified method the background of the Poisson method must first be understood.

Discovery of the Poisson Method

Simeon Denis Poisson (1781 - 1840) contributed significantly to mathematics and physics in France (Haight, 1967). Late in his life he became extremely interested in the application of mathematical probability to the administration of justice. In his major work of 1837, Poisson developed the basic probability statement utilized in his probability formula. Poisson presents his probability statement as an isolated formula and does not develop it further in that work or in any other works prior to his death (Haight, 1967). There are no references to Poisson probability between 1841 and 1898 (Haight, 1967) although other mathematicians inadvertently rediscovered and used parts of it.

Bortkiewicz (1868 - 1931) was the first to recognize similarities between the Poisson formula and certain types of discrete data (Haight, 1967, p. 115). Poisson discovered the mathematical expression; however, Bortkiewicz must be given credit for discovering the probability distribution (Haight, 1967, p. 115). Bortkiewicz first used Poisson probability in his study of the frequency of death by horse kick in the Prussian army.

Utilization of the Poisson Method

Bateman was the first to show that particle disintegration of radioisotopes could be described by a Poisson distribution (Haight, 1967, p. 120). The finding was stated in an appendix written for a scientific paper authored by Rutherford and Geiger in 1910.

Until Bateman's discovery, most researchers were trying unsuccessfully to fit radioactive disintegration directly to a normal (Gaussian) distribution. When the disintegration data was applied to a normal distribution, disintegration frequencies near zero increased when normal theory required a decreasing trend (Haight, 1967, p. 119).

Application of Poisson Method to Particle Counting

What is different about the Poisson distribution as compared to other statistical distributions? If one views a standard binomial distribution as the probability of a certain event occurring in a certain number of trials (i.e.: number of heads occurring in one flip of three coins), the Poisson distribution describes the probability of a certain event occurring in a unit of time or space (e.g.: auto crashes per month, number of defective parts in a standard batch of parts, disintegrations per second of a radioactive source). The "per unit of time or space" of a Poisson distribution is equal to the number of trials (n) of a binomial distribution.

Why can Poisson statistics and not normal statistics be used to describe radioactive disintegration? The probability of obtaining at least one disintegration of a radioactive source in one second is extremely high. Also, as described by Poisson probability, there is a tendency for many radioactive disintegrations to occur close to each other and then have a relatively long time until the next disintegration (Haight, 1967, p. 51). This can lead to

the high frequency of disintegrations at the zero point of a normal distribution that is contrary to Gaussian theory.

A situation where the unit of time is divided into many subdivisions (n is large) and there is little probability of success in a single trial (p is small) approaches a Poisson probability distribution (Chao, 1974, p. 168). The Poisson formula can be stated as follows:

$$P_{(x)} = e^{-\mu} \frac{\mu^x}{X!} \qquad (1)$$

Where: μ = mean number of successes per unit time

x = actual number of successes

Example: Assuming the true average count is 20, what is the probability of obtaining a count of 18?

$$P_{(18)} = e^{-20} \frac{20^{18}}{18 \cdot 17 \cdot 16 \cdot \ldots 1} = .0844. \qquad (2)$$

Notice that the answer is for a discrete point (i.e.: 18 counts). If the data for all possible counts around a given mean is plotted, the result is a distribution that approaches a binomial distribution and is positively skewed (Chao, 1974, p. 170). With a given probability, as n increases, (and consequently μ) the Poisson distribution approaches the symmetrical shape of a normal distribution. In fact, the normal distribution is the limit of the Poisson distribution as μ approaches infinity (Chao, 1974, p. 171).

A unique feature of the Poisson distribution, one that confuses many who try to associate the modified use of the Poisson with the normal distribution, is that the Poisson actually has only one parameter, the mean. What about the other required distribution parameter, variance? In a Poisson distribution, the variance is equal to the mean.

A simplified proof is supplied:

$$\sigma_x^2 = n\,p\,q \quad . \tag{3}$$

Where: n = number of trials

p = probability of event x occurring

q = 1 - p

As p approaches zero, (1 - p) approaches 1. Therefore:

$$\sigma_x^2 \simeq n\,p = \mu \quad . \tag{4}$$

Problem with Poisson Application to Particle Counting

In early radioactivity research, scientists such as Rutherford and Geiger used photographic plates and cloud chambers to detect radioactive decay. These devices required that the actual particle tracks be counted to obtain a number for disintegration counts. Poisson probability worked well with these methods but the methods themselves were highly inaccurate for actual count rates.

Modern detection instruments are efficient for determining count rates; however, certain counting instruments (e.g.: proportional counters and Geiger Muller detectors) have a phenomenon called dead time, which represents a period of time (several hundred microseconds) in which the detector is inoperative after an ionizing event. This dead time may be of constant or stochastic length and will lead to considerable error when Poisson statistics are applied to counting events.

In the measurement of radioactive disintegration, the possibility exists of events being arbitrarily near each other. Since the Poisson is a negative

exponential function, its gap distribution (Haight, 1967, p. 51) suggests that these disintegration events are more likely to be close together than far apart. If this is true, the counter is likely to miss some events and the distribution derived from this counter will deviate from a true Poisson distribution.

It is mathematically feasible to calculate a correction formula (Feller, 1948) for this phenomenon. However, this formula is difficult to use in normal counting situations. The formula is also much more exacting than is justified in view of standard instrument efficiency.

Modification of Poisson to Gaussian Distribution

A much simpler and more effective solution suggests that the limit of the Poisson distribution as μ approaches infinity ($\mu \geq 20$ approaches a normal distribution) is a normal (Gaussian) distribution (Chao, 1974). In this approach, the basic concepts and parameters of the Poisson are used while the evaluation processes normally associated with the Gaussian distribution are employed (Chase/Rabinowitz, 1967).

Field Application of the Modified Distribution

The basic fact that must be borrowed from Poisson statistics is that the mean is equal to the variance (Chase/Rabinowitz, 1967, p. 83). Keeping this in mind, derivation of some counting terms in relation to the Poisson mean may be achieved (Cember, 1969). Counting rate and counting time may be expressed in terms of the mean such that:

$$\sigma_g^2 = \mu_g = \frac{r_g}{t_g} \quad . \qquad (5)$$

Where: r_g = gross count rate in counts per minute (cpm)

t_g = gross count time in minutes

In the same manner, one may express the same relationship for background radiation count:

$$\sigma_b^2 = \mu_b = \frac{r_b}{t_b} \quad . \qquad (6)$$

Cember (1969) suggests a relationship between sample and background variance that will allow determination of an interval estimate at a given confidence level. The relationship is as follows:

$$\sigma_n = \sqrt{\sigma_g^2 + \sigma_b^2} = \sqrt{\frac{r_g}{t_g} + \frac{r_b}{t_b}} \quad . \qquad (7)$$

Where: σ_n = standard error of the mean for net counting rate

In determining whether two samples are significantly different, one cannot rely upon subjective opinion. However, the concept of interval estimation may be employed to assess with a stated level of confidence whether two samples are significantly different or not. An interval estimate is generally stated as (Mendenhall & Ott, 1980):

$$\bar{Y} \pm z\,\sigma_{\bar{y}} \quad . \qquad (8)$$

Where: $\bar{Y}$ is the sample mean
z is the confidence coefficient
$\sigma_{\bar{y}}$ is the standard error of the mean

Using the concept of interval estimation at a desired confidence level of 95% (.05 level of significance) equation 8 takes the form:

$$\bar{Y} \pm 1.96 \sqrt{\frac{r_g}{t_g} + \frac{r_b}{t_b}} \quad . \qquad (8a)$$

That is, if two samples vary by more than $1.96\sigma_{\bar{y}}$, one can say that they are significantly different with 95% confidence.

Interval estimation may be used in a slightly different manner to determine whether or not a sample is truly radioactive. This method requires evaluation of the sample in terms of the null hypothesis.

For the case of determining whether or not the particular sample is radioactive, the researcher will state the null hypothesis (H_o) in terms of background radiation; that is, the average count of the sample is equal to the normal fluctuation of background count rate ($H_o: \mu_s = \mu_b$). With a level of significance of .05, this two tailed evaluation of H_o modifies equation 8a to:

$$\bar{Y} \pm 1.96 \sqrt{\frac{r_b}{t_s} + \frac{r_b}{t_b}} \quad . \qquad (9)$$

Where: t_s = time of sample count

If background radiation is viewed as the actual mean of the background distribution, any variance of the sample below background may be rejected and evaluation of H_o in terms of a one tailed distribution, $\mu_s > \mu_b$, (i.e.: only count rates greater than background) is possible. A one tailed evaluation has the distinct advantage of allowing the same level of significance while using a numerically lower confidence coefficient (Cember, 1969, p. 276).

Equation 9 may now be written:

$$\bar{Y} \pm 1.64 \sqrt{\frac{r_b}{t_s} + \frac{r_b}{t_b}} \quad . \qquad (10)$$

Reconstructing the interval estimate in terms of positive background count rates only and correcting the sample count rate for background, a simplified expression for the minimum detectable counts of the sample results:

$$MDCS = 1.64 \sqrt{\frac{r_b}{t_s} + \frac{r_b}{t_b}} \quad . \qquad (11)$$

Equation 11 now gives a threshold count rate above background that shows a sample as being radioactive with a confidence level of 95%. Notice that the level of significance (.05) has not changed, but the confidence coefficient has decreased from 1.96 to 1.64 through the analysis of the null hypothesis as a one tailed distribution. Equation 11 allows the laboratory technician

to evaluate a sample as radioactive with a lower count rate above background than would otherwise be possible and still maintain the same level of confidence.

Summary

It is the writer's contention that radioactive disintegration is best described statistically by a Poisson distribution. In actual practice, the inefficiencies of counting instruments, namely dead time, render the Poisson ineffective in its pure state. Gap distribution, a mathematical correction for dead time, may be utilized. However, it is impractical from a field use standpoint.

A simpler solution is to utilize the limit of the Poisson distribution, the Gaussian distribution, to describe the radioactive disintegration. This approach is valid assuming that the Poisson relationship, $\mu = \sigma^2$ is applied in the derivation of working formulas.

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